Mathematical Modelling with Fully Anisotropic Diffusion

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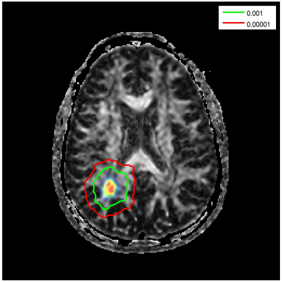

Fully Anisotropic Diffusion

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- Where does this model come from?
- What are mathematical properties?

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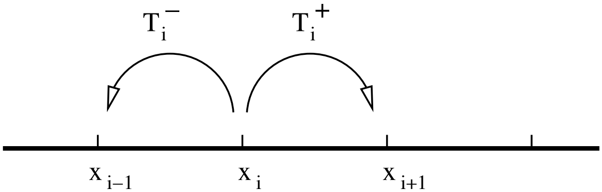
- Where does this model come from?
- What are mathematical properties?
- What can it be used for?

Outline

(1.1) Random walk



$u_i(t)$: Probability to find a particle at x_i at time t.
$T_i^{\pm}$: Transitional probabilities

Master equation:

$$\frac{du_i}{dt} = T_{i-1}^{+} u_{i-1} + T_{i+1}^{-} u_{i+1} - \left(T_i^{+} + T_i^{-}\right) u_i .$$

Three cases

1. look locally: $T_i^{\pm}(x) = (\Delta x)^{-2}\ T(x_i)$

$$u_t \approx (Tu)_{xx}$$

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$$u_t \approx (Tu_x - T_x u)_x$$

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2. look ahead: $T_i^{\pm}(x) = (\Delta x)^{-2}\ T(x_{i\pm 1})$

$$u_t \approx (Tu_x - T_x u)_x$$

3. look ahead half way $T_i^{\pm}(x) = (\Delta x)^{-2}\ T(x_{i\pm 1/2})$

$$u_t \approx (Tu_x)_x$$

Othmer, Stevens 1997, Okubo, Levin 2002

(1.2) Ideal free distribution

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- Let $\mu(x)$, $x \in \mathbb{R}$ describe the resource landscape.
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- Let $\mu(x)$, $x \in \mathbb{R}$ describe the resource landscape.
- The population $u(x)$ has an ideal free distribution, if $u \sim \mu$.
- If individuals move faster in bad resources, such as

$$D(x) = \mu(x)^{-1}$$

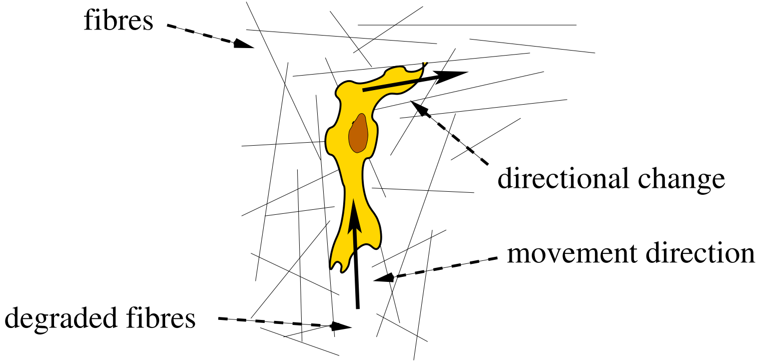
then the ideal free distribution is realized by steady states of

$$u_t = (D(x)u)_{xx} + ru(\mu(x) - u).$$

C. Cosner, S. Cantrell, M Lewis et al.

Outline

(1.3) Cell movement in fiber networks



H', JMB, 2006

Experiments of Friedl and Wolf
(fibrosarcoma HT1080/MT1-MMP cells):

Movie: JCB21

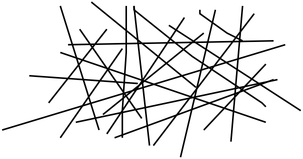
Tumor spheroid

Movie: NCB31

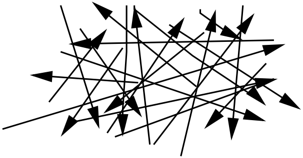
A model for mesenchymal motion, (H', JMB 2006)

Fibre orientation: $\theta \in S^{n-1}$.

Undirected fibres (collagen), $\theta \approx -\theta$.



Directed fibres (micro tubules or actin filaments), $\theta \neq -\theta$.



Fibre Directional Distribution

- Orientation $\theta \in S^{n-1}$.
- Distribution of fibres $q(t, x, \theta)$

$$\int_{S^{n-1}} q(t, x, \theta) d\theta = 1.$$

Cell Velocities

Set of all possible cell velocities V:

$$V = [s_1, s_2] \times S^{n-1}, \qquad 0 \le s_1 \le s_2 < \infty.$$

$$\hat{v} := \frac{v}{\|v\|}$$

Distribution on V

q is a distribution on S^{n-1}.
To make this into a distribution on V we consider

$$\frac{q(t,x,\hat{v})}{\omega}$$

where

$$\omega = \int_V q(t,x,\hat{v})dv = \begin{cases} \frac{s_2^n - s_1^n}{n}, & s_1 < s_2, \\ s^{n-1} & s_1 = s_2 = s. \end{cases}$$

Transport Equation

$p(t,x,v)$: cell distribution at time t, location x, velocity v.

$$\begin{aligned} p_t(t,x,v) + v\cdot\nabla p(t,x,v) &= -\mu p(t,x,v) \\ &\quad +\mu\int_V \frac{q(t,x,\hat{v})}{\omega} p(t,x,v')dv' \end{aligned}$$

$\mu > 0$ constant turning rate.
$\frac{q(t,x,\hat{v})}{\omega}$: probability distribution of new chosen directions.

Transport Equation

$$p_t + v \cdot \nabla p = \mu \left(\frac{q}{\omega} \bar{p} - p \right)$$

$$\bar{p} = \int_V p(t, x, v) dv.$$

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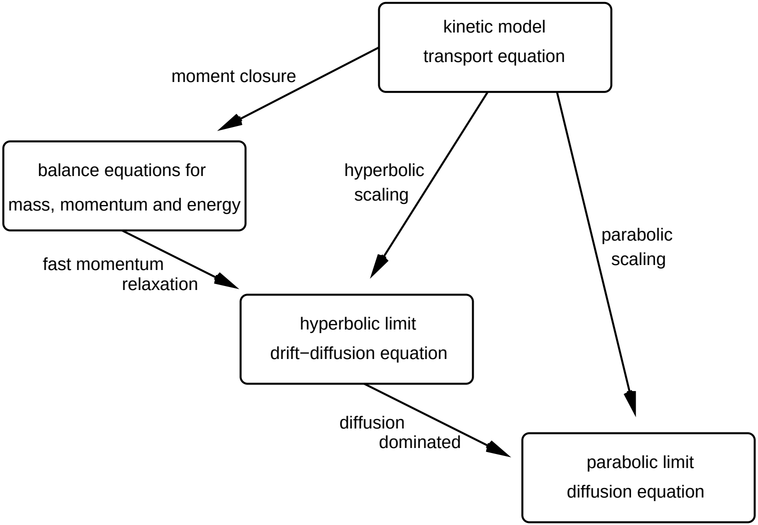
Plus an equation for the tissue changes

$$q_t(t, x, v) = G(v, q, p).$$

Drift Diffusion Limits



H', Painter, *Kinetic Models for Movement in Oriented Habitats and Scaling Limits*, to appear 2012

Hyperbolic Scaling for Directed Tissue

$$\tau = \varepsilon t, \qquad \xi = \varepsilon x,$$

$$\varepsilon p_\tau + \varepsilon v \cdot \nabla_\xi p = \mathcal{L} p$$

$$\mathcal{L} p = \mu \left(\frac{q}{\omega} \, \bar{p} - p \right)$$

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Use method of Chapman Enskog expansion.

Hyperbolic Scaling

$$
\begin{aligned}
\bar{p}_\tau + \nabla \cdot (u_c \bar{p}) &= 0. \\
u_c(t,x) &= \int_V v \frac{q(t,x,\hat{v})}{\omega} dv = \beta < q >
\end{aligned}
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The drift velocity u_c is the mean value of q/ω over V.
Note that $u_c = 0$ for undirected tissue.

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Use method of regular expansion in ε.

Parabolic scaling (diffusion dominated)

$$\begin{aligned} \bar{p}_\tau &= \nabla(\nabla(D\bar{p})) \\ D(t,x) &= \frac{1}{\mu}\int_V v\, v^T \frac{q(t,x,\hat{v})}{\omega}\, dv \end{aligned}$$

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\end{aligned}
$$

D is the variance-covariance matrix of $q(t,x,\theta)$.

Moment closure (mixed case)

Moment closure with fast momentum relaxation:

$$\bar{p}_t + \nabla \cdot (u_c \bar{p}) = \nabla(\nabla D \bar{p})$$

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Procedure:

1. Define $q(t, x, \theta)$ based on biological insight
2. Compute mean and variance of q/ω:

$$\begin{aligned} u_c &= \int v\, q/\omega\, dv \\ D &= \int (v - u_c)(v - u_c)^T\, q/\omega\, dv \end{aligned}$$

3. Use drift-diffusion model

Example: aligned and undirected tissue

Assume

$$q(t,x,\theta) := \begin{cases} 0.5 & \text{for } \theta = e_1, \\ 0.5 & \text{for } \theta = -e_1, \\ 0 & \text{otherwise.} \end{cases}$$

and assume that $V = sS^{n-1}$.

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and assume that $V = sS^{n-1}$.

Then the drift diffusion limit is

$$\bar{p}_t = \frac{s^2}{\mu} \bar{p}_{x_1 x_1}$$

Outline

Brain tumors

[*Swanson 2000*]

$$u_t = \nabla(D(x)\nabla u) + \rho u$$

$$D(x) = \begin{cases} 5 & x \in \{\text{white matter}\} \\ 1 & x \in \{\text{gray matter}\} \end{cases}$$

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$$D(x) = \begin{cases} 5 & x \in \{\text{white matter}\} \\ 1 & x \in \{\text{gray matter}\} \end{cases}$$

However

$$D(x) = \frac{1}{\omega}\int_V vv^T q(x, \hat{v})\, dv$$

is the variance-covariance matrix. Swansons assumption does not reflect the fibrous structure of white matter.

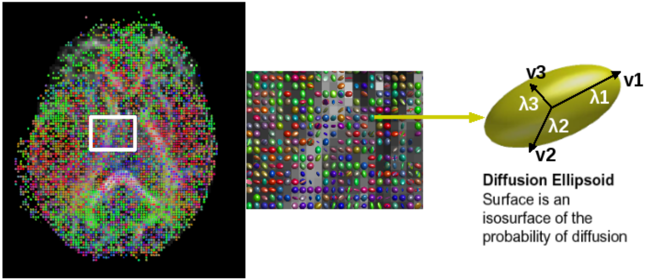
White Matter

If at a point $x \in$ white matter we find a dominant direction of e_1, say, then we would expect

$$D(x) = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

DTI - diffusion tensor imaging

[Jägersand, Murtha, Beaulieu, 2000s]



DTI with tensors represented as ellipsoids

Diffusion Ellipsoid
Surface is an isosurface of the probability of diffusion

Tumor diffusion tensor

- Water diffusion tensor $D_w \in \mathbb{R}^{3\times 3}$:

$$D_w = \lambda_1 v_1 v_1^T + \lambda_2 v_2 v_2^T + \lambda_3 v_3 v_3^T$$

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- [*Jbabdi, Swanson et al. 2005*]:
  Tumor diffusion tensor

$$D_T = a_1(r)\lambda_1 v_1 v_1^T + a_2(r)\lambda_2 v_2 v_2^T + a_3(r)\lambda_3 v_3 v_3^T,$$

  where r is a scaling parameter.

Jbabdi et al. 2005, Figure 1

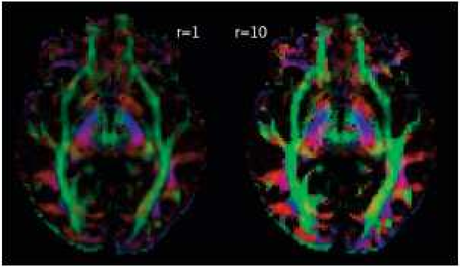

FIG. 1. RGB color maps of the tensor's principal diffusion directions. Left: tensor with no change in tumor cell diffusion anisotropy compared to water anisotropy (r = 1). Right: Tensor with change in tumor cell anisotropy (r = 10).

Jbabdi et al. 2005, Figure 3

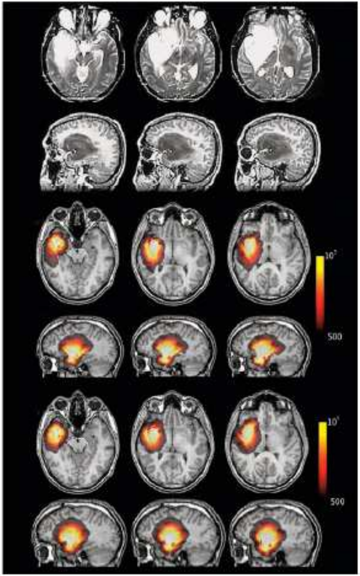

FIG. 3. Simulations with a starting point located in the insular part of the uncinate fasciculus. (a) Patient data. (b) Anisotropic simulations (r = 10). (c) Isotropic simulations. Visualization threshold : 500 cells per mm³.

Research stimulated by Swanson, Jbabdi et al.

Question:

How to relate DTI information to tissue structure and to tumor diffusion?

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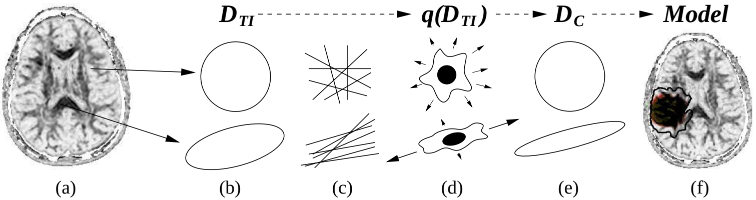
How to relate DTI information to tissue structure and to tumor diffusion?

Possible answer:

Using transport equation framework and the von-Mises distribution.



von Mises distribution

Let $\gamma \in S^{n-1}$ be a given direction. Define the von Mises distribution

$$q(x,\theta) = \frac{1}{2\pi I_0(k)} e^{k(x)\theta\cdot\gamma},$$

where $k(x)$ is the parameter of concentration.

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- q becomes uniform for $k \to 0$.
- q becomes a singular delta-distribution in direction γ for $k \to \infty$.

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Recall

$$u_c = \int v \frac{q}{\omega} dv, \quad D = \int (v - u_c)(v - u_c)^T \frac{q}{\omega} dv.$$

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Recall

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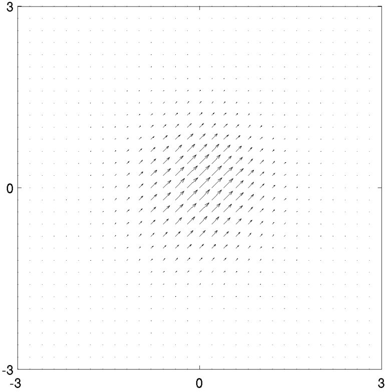
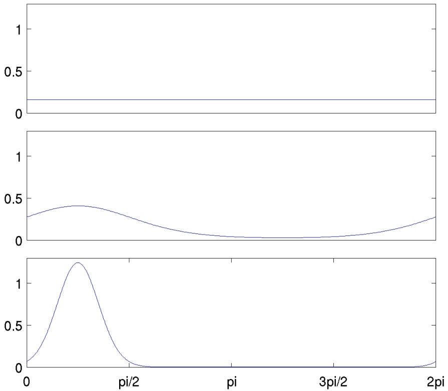
For the advection-diffusion limit in 2D with $V = sS^1$:

$$\begin{aligned} u_c &= s\frac{I_1(k)}{I_0(k)}\gamma \\ \frac{2\mu}{s^2} D &= \left(1 - \frac{I_2(k)}{I_0(k)}\right)\mathcal{I} + \left(\frac{I_2(k)}{I_0(k)} - \left(\frac{I_1(k)}{I_0(k)}\right)^2\right)\gamma\gamma^T \end{aligned}$$

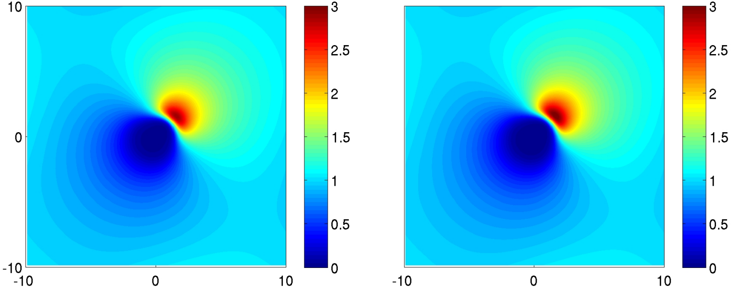
Example 2: Oriented





Solution of

$$u_t = \nabla\nabla(D(x)u)$$



(A) $s = 10, \mu = 100$, (B) parabolic limit.

Bimodal von-Mises distribution

Let $\gamma \in S^1$ be a given direction. Define the bimodal von Mises distribution

$$q(x,\theta) = \frac{1}{4\pi I_0(k)} \left(e^{k(x)\theta\cdot\gamma} + e^{-k(x)\theta\cdot\gamma} \right),$$

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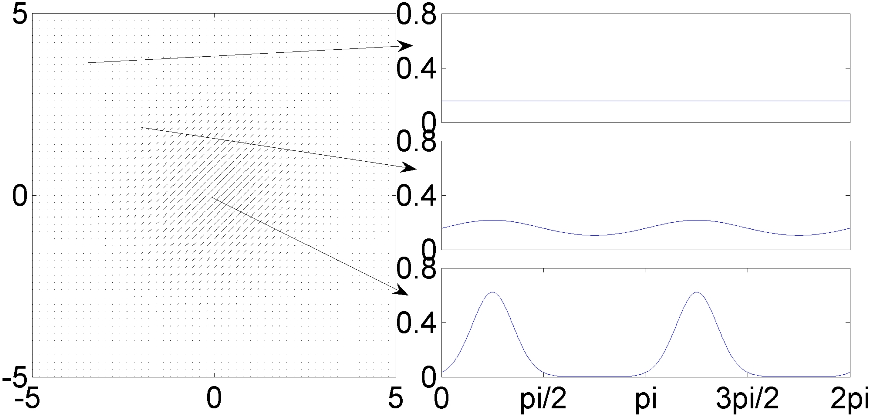
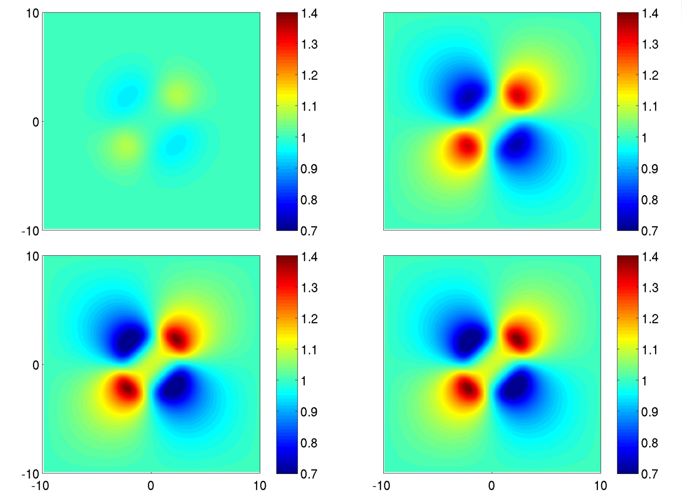
where $k(x)$ is the parameter of concentration.
For the advection-diffusion limit:

$$\begin{aligned} u_c &= 0 \\ \frac{2\mu}{s^2} D &= \left(1 - \frac{I_2(k)}{I_0(k)}\right) \mathcal{I} + \frac{I_2(k)}{I_0(k)} \gamma\gamma^T \end{aligned}$$

Example 2: Nonoriented





(A) $s = 0.1, \mu = 0.01$, (B) $s = 1, \mu = 1$, (C) $s = 10, \mu = 100$
(D) parabolic limit.

Application to DTI data

- Water diffusion tensor D_{TI} with eigenvectors and eigenvalues (λ_i, e_i).
- dominating direction $\gamma := e_1$
- Fractional anisotropy

$$\mathrm{FA}(D_{TI}) := \frac{\sqrt{(\lambda_1 - \lambda_2)^2 + (\lambda_2 - \lambda_3)^2 + (\lambda_1 - \lambda_3)^2}}{\sqrt{2(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)}}$$

- concentration coefficient

$$k := \kappa \mathrm{FA}(D_{TI}), \qquad \kappa > 0.$$

Fractional Anisotropy

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- $D_1 = \mathrm{diag}(1,1,1)$, then $\mathrm{FA}(D_1) = 0$.
- $D_2 = \mathrm{diag}(1,1,0)$, then $\mathrm{FA}(D_2) = 1/\sqrt{2}$.
- $D_3 = \mathrm{diag}(1,0,0)$, then $\mathrm{FA}(D_3) = 1$

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- $\mathrm{FA}(D) \in [0,1]$

Procedure

1. Obtain measured DTI data

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2. Define $q(x)$ as von-Mises distribution with dominating direction $\gamma = e_1$ and concentration parameter $k = \kappa \mathrm{FA}$. Use $\kappa > 0$ as adjustable parameter.

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3. Compute tumor diffusion tensor as second moment of $q(x)$:

$$D(x) = \int vv^T q(x)/\omega dv.$$

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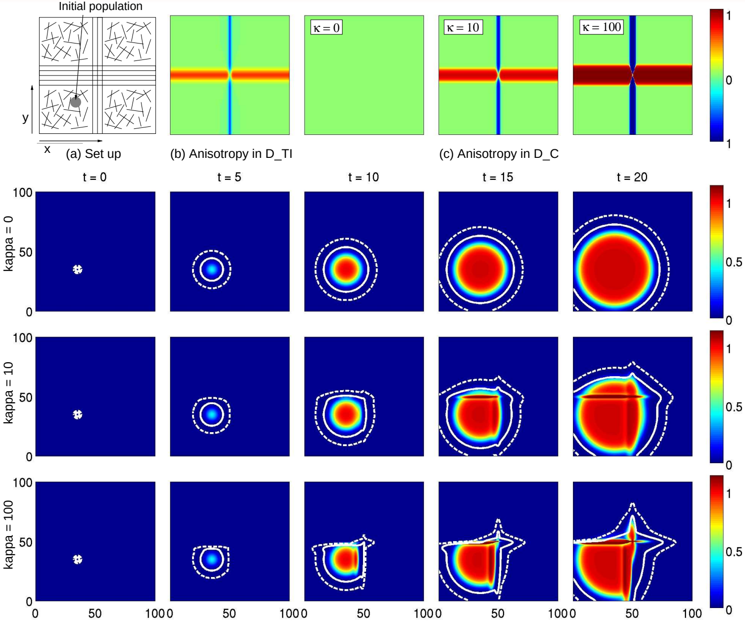
2. Define $q(x)$ as von-Mises distribution with dominating direction $\gamma = e_1$ and concentration parameter $k = \kappa \mathrm{FA}$. Use $\kappa > 0$ as adjustable parameter.
3. Compute tumor diffusion tensor as second moment of $q(x)$:
$$D(x) = \int vv^T q(x)/\omega dv.$$
4. Solve fully anisotropic diffusion equation on brain domain
$$u_t = \nabla\nabla(D(x)u).$$

Artificial brain



(d) Glioma growth for varying kappa

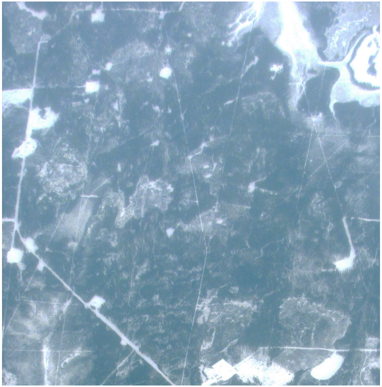
Using Anisotropic Diffusion for Brain Tumors

K.J. Painter:

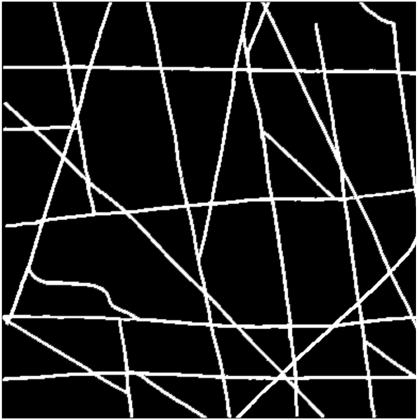
growth.avi

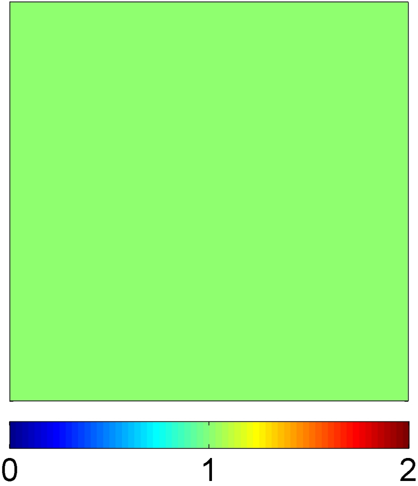
Outline

Wolf Movement on Seismic lines

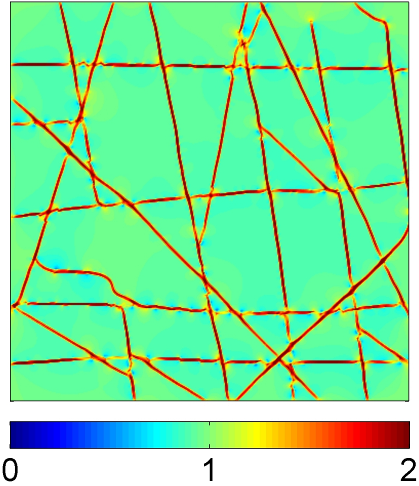


McKenzie, Lewis et al. 2011

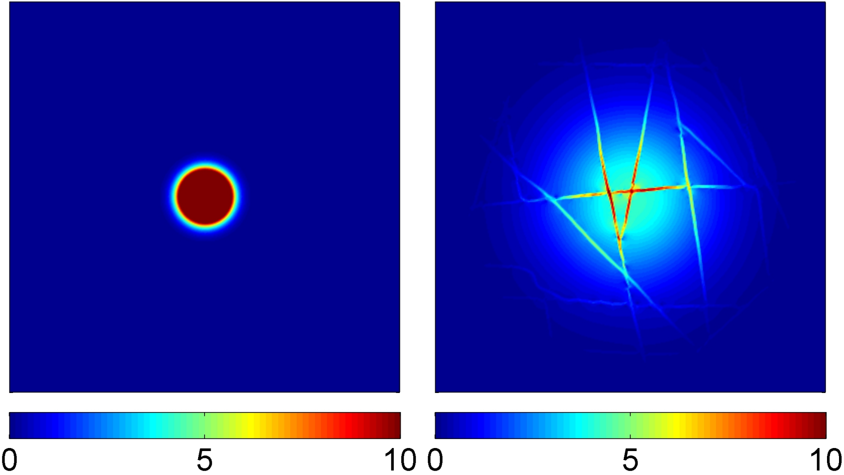




initial condition



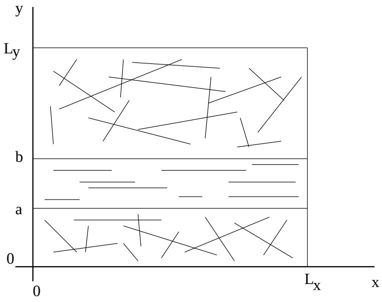
after time $t = 10$



initial conditions after time $t = 5$

Outline

Example of a road or white matter track



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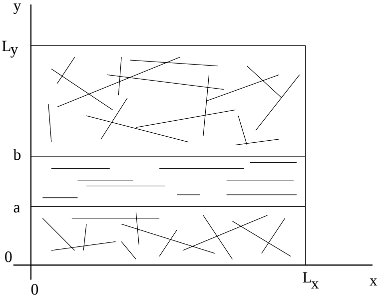

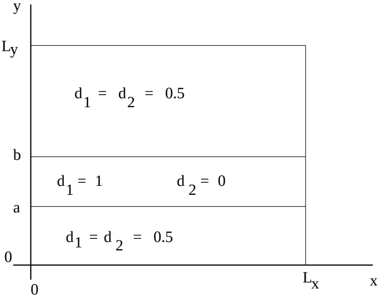

D is diagonal and

$$u_t = (d_1(x,y)u)_{xx} + (d_2(x,y)u)_{yy}$$

- regular diffusion: $d_1 = d_2 =$const.
- diffusion biased in x direction: $d_1 >> d_2$.

A Mathematical problem

On $\Omega = (0, L_x) \times (0, L_y)$ consider

$$u_t = (d_1(x, y)u)_{xx} + (d_2(x, y)u)_{yy}$$

under no-flux boundary conditions and

$$d_1 = d_2 = 1/2, \qquad \text{for} \quad y \notin (a, b),$$

$$d_1 = 1, d_2 = \varepsilon \qquad \text{for} \quad y \in (a, b).$$

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Goal: Understand behavior as $\varepsilon \to 0$.

Blow-up result

Theorem (H', Painter, Winkler, 2012)
Assume $\varepsilon = 0$. Let

$$U(y,t) = \int u(x,y,t)dx.$$

Then in the sense of regular Borel measures over $[0, L_y]$ we have

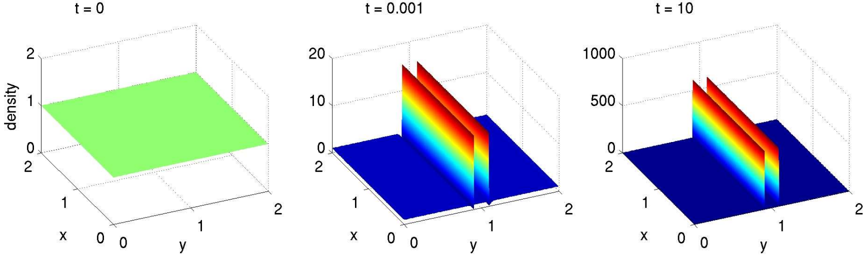
$$U(y,t) \rightharpoonup^* \frac{1}{L_x}\chi_{(a,b)}(y)U_0(y) + m_1\delta(y-a) + m_2\delta(y-b),$$

as $t \to \infty$, where

$$m_1 = \int_{\{y<a\}} u_0 dxdy, \qquad m_2 = \int_{\{y>b\}} u_0 dxdy.$$

Simulations

(K.J. Painter)



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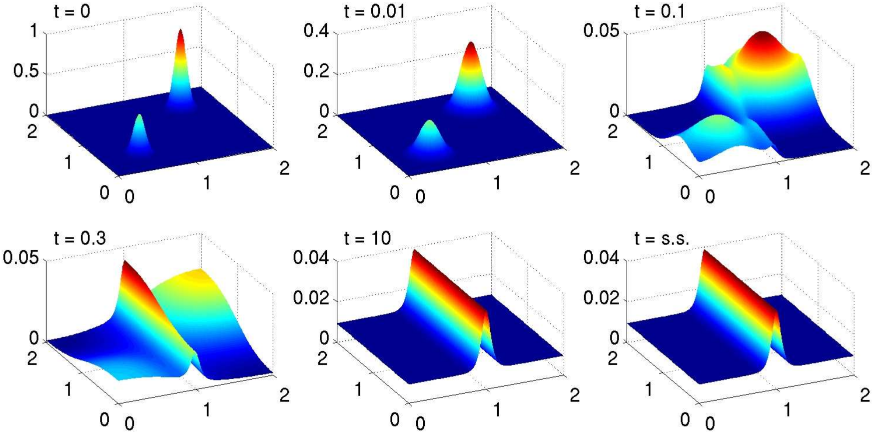

(a) epsilon = 0.1

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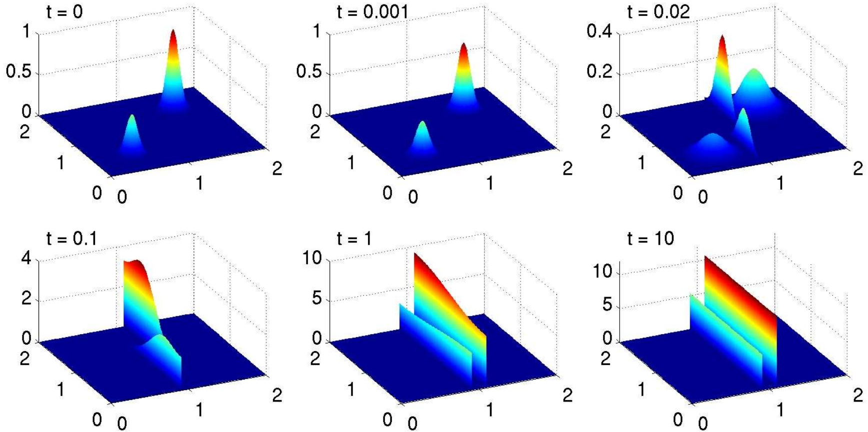

(b) epsilon = 0.001

Outline

(1) Model derivations

(2) Mesenchymal motion

(3) Brain Tumors

(4) Wolf Movement

(5) A Blow-up result

(6) Conclusions

Conclusions

Fully anisotropic diffusion:

- Where does it come from?
  random walk, ideal free distribution, transport equations

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  random walk, ideal free distribution, transport equations
- What are mathematical properties?
  pattern formation and blow-up
- What can it be used for?
  glioma growth and wolf movement

Open Problems

Glioma:

- Fully develop 3D model

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Math:

- Pattern formation and travelling waves
- Uniqueness of very weak solutions
- Other blow-up?

References

- H', J. Math. Bio., 2006
  model development, drift-diffusion limits
- Painter, J. Math. Bio., 2009
  extension to fibre production, simulations
- Chauviere, H', Preziosi, NHM 2007 and DCDS-B 2007
  inclusion of cell-cell, cell-ECM interactions and chemotaxis
- H', Hinow, Wang, 2009, DCDS-B
  existence of mild solutions and weak steady states
- H', Painter, book chapter, 2012
  anisotropic diffusion models, wolf movement
- H', Painter, Winkler, 2013, European J. Appl. Math.
  Blow-up result
- Painter, H' submitted 2012
  application to glioma