Courtesy from Pixels et Grains d'argent

# A plant pathogen multiscale problem

## dedicated to Chris Cosner

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# Multi-scale problem

*Courtesy from Pixels et Grains d'argent, and from INRA*

# *Host-parasite system*

- *Vitis vinifera*, vine stock & plot,
  - ***Spatial structures :*** *leaves, plants, plots*, ***no dispersal***,
  - ***Seasonal variations :*** *growing season, climate (temperature),*
  - ***Highly anthropized system*** : wide diversity of cropping systems driving *host dynamics (secondary shoots / leaves appearance),*
  - No specific resistance of the host.

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  - ***Highly anthropized system*** : wide diversity of cropping systems driving *host dynamics (secondary shoots / leaves appearance),*
  - No specific resistance of the host.

- *Erysiphe necator*, powdery mildew,
  - ***Airborne dispersal of conidia (spores),***
  - ***Sensitive to the climate*** : *wind speed & direction* (*?*),
  - ***Sensitive to the quality of its host :*** e.g., *age of leaves*, …
  - Hard to detect and quantify at the vineyard level,
  - No reliable prevention tool : routine chemical sprays,
    - ❖ 30% world market in fungicides dedicated to vine, (F: 60 M€/year ),
  - Over-wintering in the bark of vine stocks & within dormant buds,

# Outline : multi-scale problem

***(1) Architectural discrete 3D*** plant - pathogen model, discrete w.r.t. time & space, ***over a single growing season***,

$$X(n+1) = F(X(n), n, \pi(n), \theta(n)), \quad n \geq 0.$$

***(2) Continuous aggregated*** plant - pathogen model, ODEs SEIRT-like, ***for a perennial plant***,

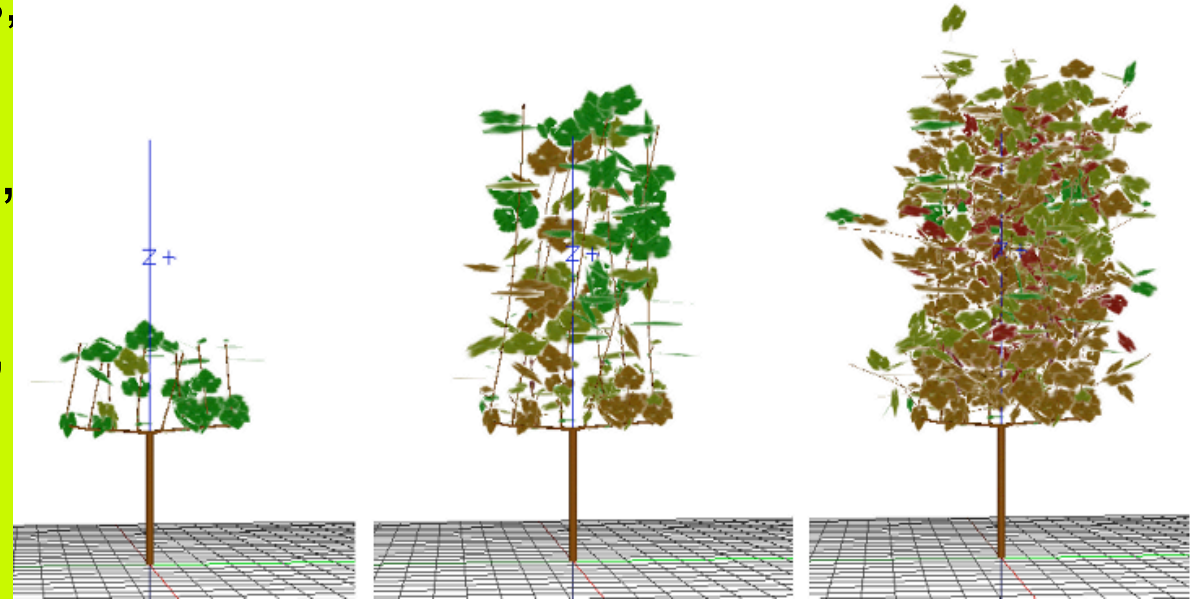
$$d_t X(t) = F^{year}(X(t), t, \pi(t)), \quad t \geq 0.$$

***(3) Spatially structured*** plot - pathogen model, RD - SEIR(T) like,

$$\partial_t U = \nabla \cdot \left(D(x,N)\nabla U\right) + V \cdot \nabla U + G^{year}(U, x, \pi(t)), \quad t \geq 0, x \in D.$$

# *Discrete 3D*
# plant - pathogen scale

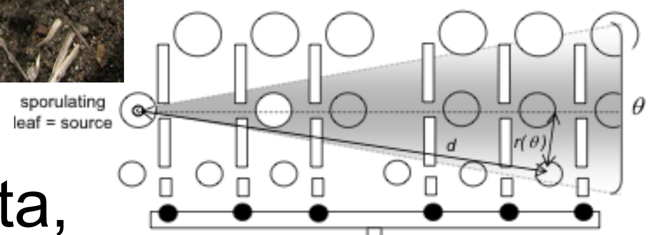
- ***Host dynamic***: complex 3D architectural model,
- Appearance, growth, size, age, 3D localization and surface area of primary,
  - Leaves, internodes,
  - Shoots, grapes,
- ***Climatic scenario***,
  - Daily basis,
- Secondary organs,
- ***Anthropization***,
  - Shoot topping,



# ***Discrete 3D***
# plant - pathogen scale

- ***Coupled host-pathogen dynamic*** : complex 3D architectural model,

- Primary infection :
  - Calendar date,
  - 3D localization on host,
- Lesion growth,
  - Latency period,
  - Spore production period,
  - Spore release,
- *Within* vine stock spore dispersal,
  - Speed of wind (daily),
- Secondary infections,



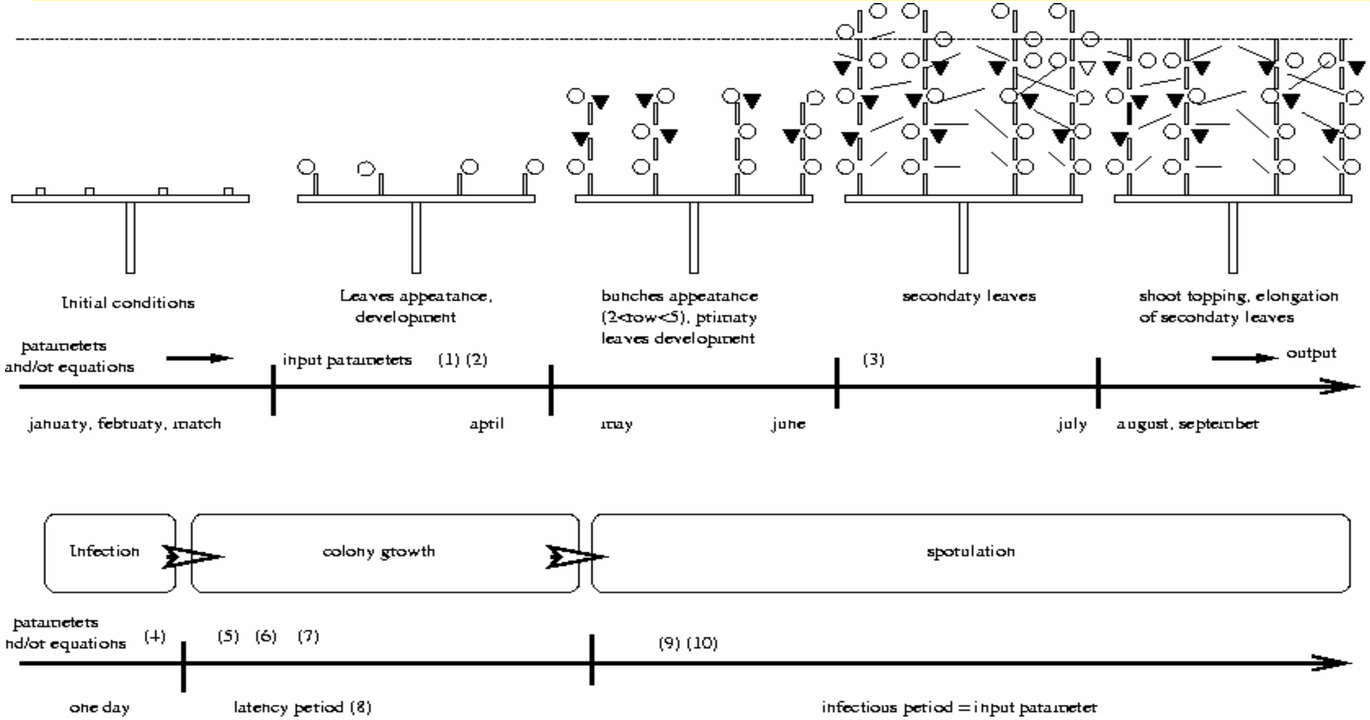

Field data,
INRA Cabernet Sauvignon,
Digitalization of vine stocks,

# *Discrete 3D*
# plant - pathogen scale

- Plant / pathogen dynamics for a growing season,

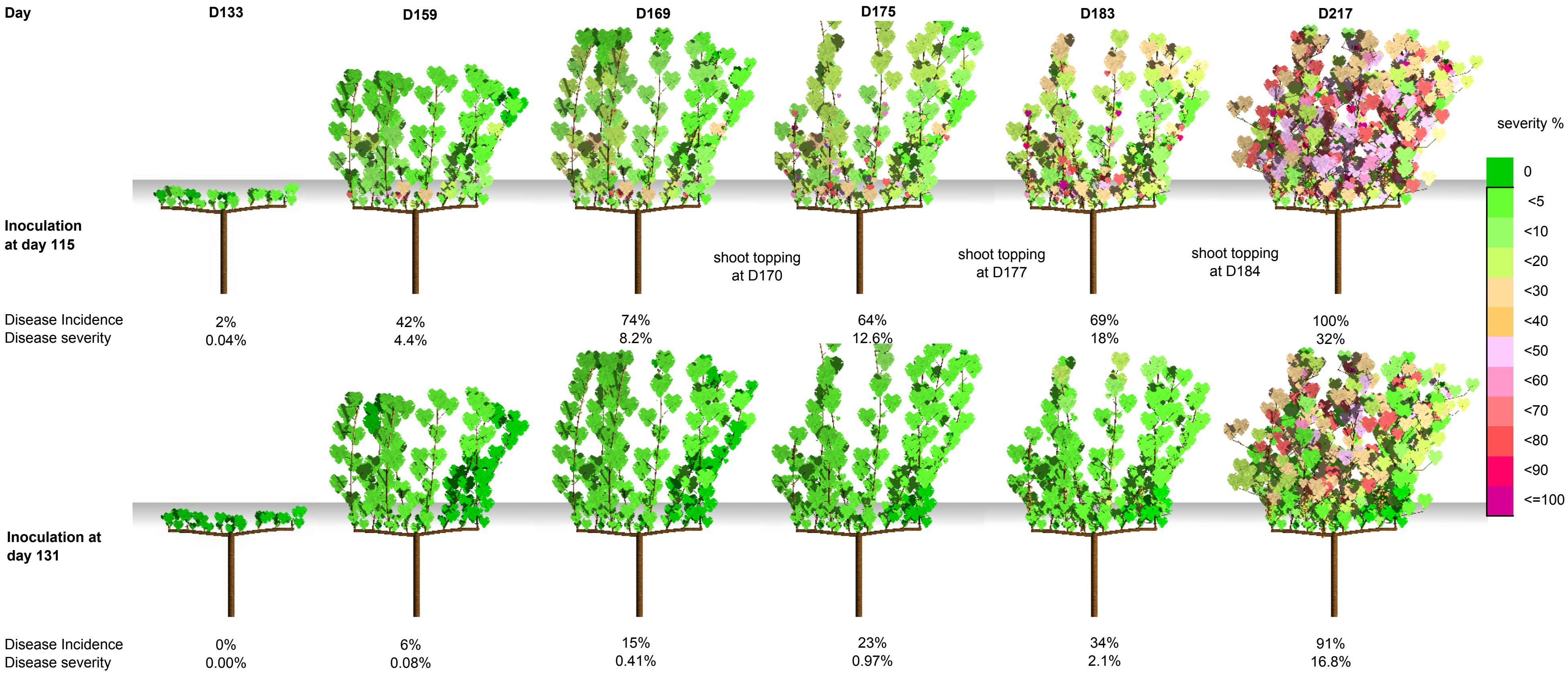
  complex 3D model, more than 30 parameters …

$$X(n+1) = F(X(n), n, \pi(n), \theta(n)), \quad n \geq 0.$$



# ***Discrete 3D***
# plant - pathogen scale

- Scenario 1998, inoculation day : 115 vs 131,



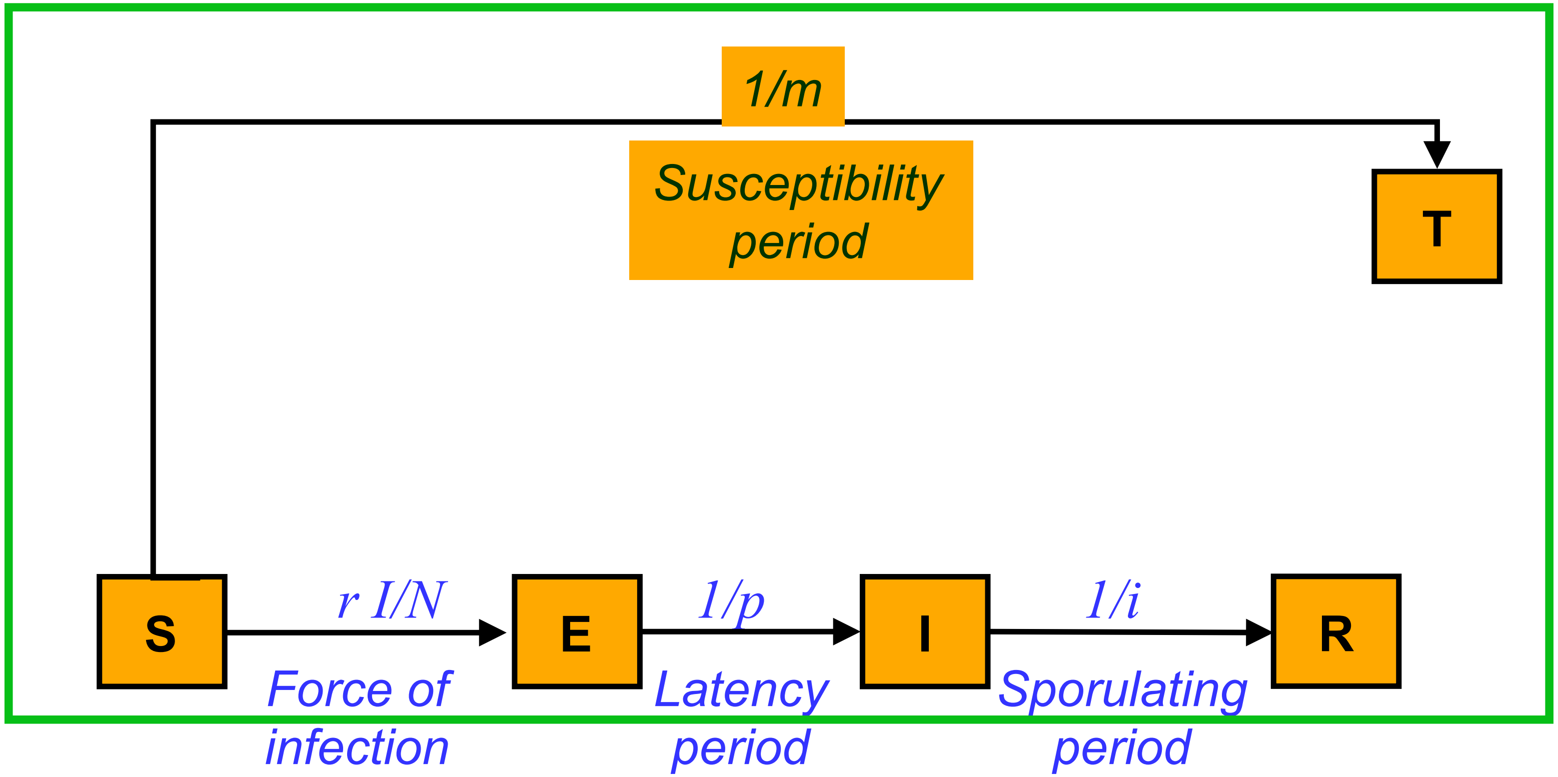

# *Discrete 3D*
# plant - pathogen scale

- PCA PLS analysis, Climatic scenarios and crop management do not impact the same variables,
  - Year (climate) is better correlated to early disease,
  - Crop management is more correlated to later disease,
- Sensitivity and elasticity analysis,
  - Age of leaves : susceptibility, ontogenic resistance,
  - Vigour of the host,
  - Sporulation,
  - Inoculation date,
  - Temperature, dominant wind, …
- Qualitative analysis ?

# ***Continuous aggregated*** plant - pathogen “minimal” model

- SEIRT compartmental model w.r.t. leaf surface area,

Host growth




  – incidence : $r \times S \times I/N$, Vanderplank, etc.

# ***Continuous aggregated*** plant - pathogen "minimal" model

- SEIRT model : ***S***usceptible, ***E***xposed, ***I***nfectious, ***R***etired, ***T*** (on***t***ogenic resistance),

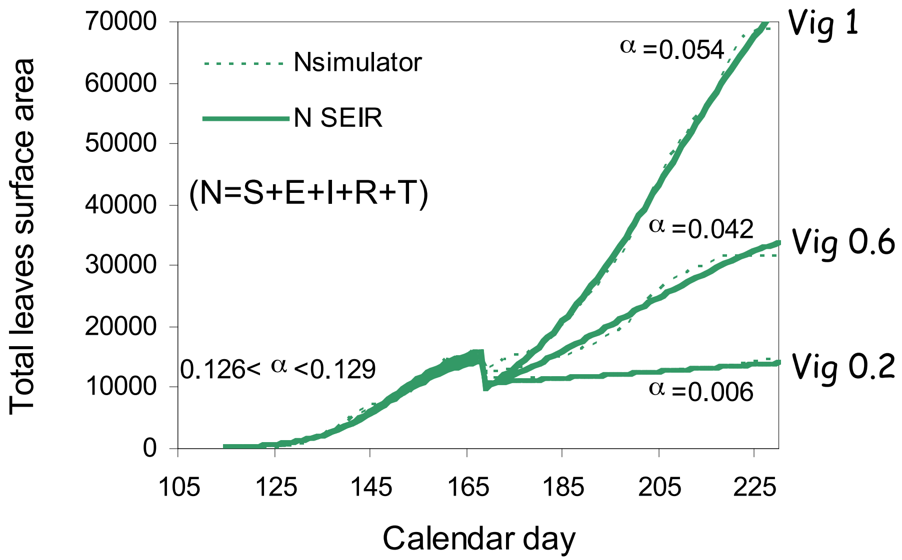
$$\begin{cases} d_t S = \Lambda - r I \frac{S}{N} - \frac{1}{m} S \\ d_t E = + r I \frac{S}{N} - \frac{1}{p} E \\ d_t I = + \frac{1}{p} E - \frac{1}{i} I \\ d_t R = + \frac{1}{i} I \\ d_t T = + \frac{1}{m} S \end{cases} \Rightarrow d_t N = \Lambda = \alpha \left( 1 - \frac{N}{K} \right) N.$$

  – Find a set of parameters $(r, \alpha, K)$ yielding consistent dynamics w.r.t. the discrete model ?

# ***Continuous aggregated*** plant - pathogen "minimal" model

- « Aggregated » ODEs model fitted with discrete model,
  - Good fitting ***to aggregate time-dependent processes*** provided, ≠ sets of parameters $(r, \alpha, K)$ at shoot topping,
  - LSM : total leaf / diseased surface area,
  - Problem at shoot topping : status of remaining leaves ?

1998



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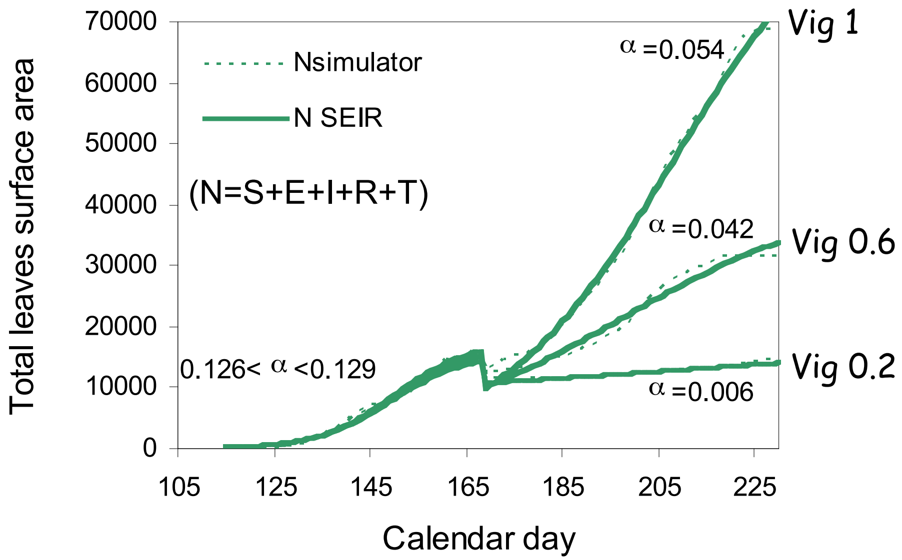


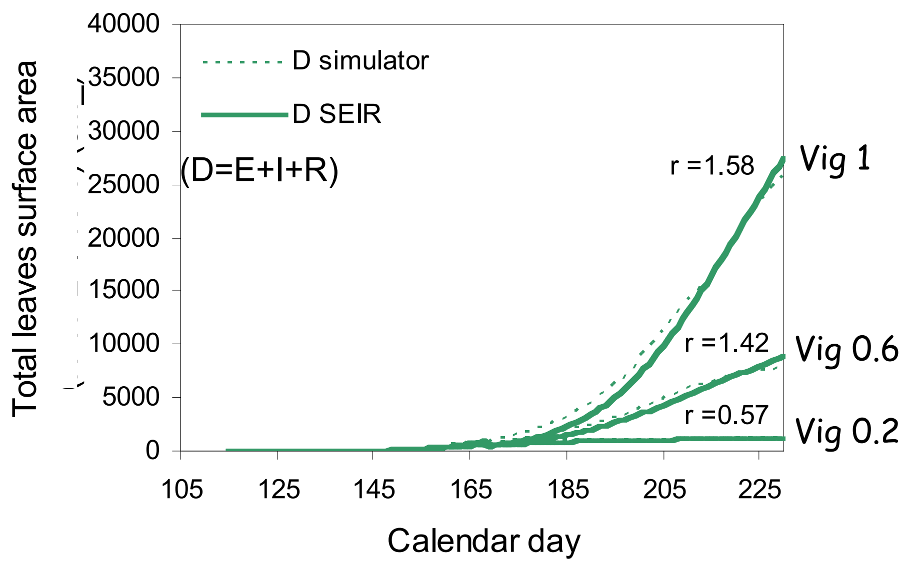

# ***Continuous aggregated*** plant - pathogen "minimal" model

- Qualitative analysis wrt standard epidemiology,
  - Computing the reproductive number $R_0$, $R_0 = r \times i$,
  - Computing the effective reproductive number $R_{\text{effective}}$,
    - ❖ $R_{\text{effective}} = r\ i\ S(t)/N(t)$,
    - ❖ $t \rightarrow R_{\text{effective}}(t)$ « decreasing outside shoot-topping » ?

# ***Continuous aggregated*** plant - pathogen “minimal” model

- Qualitative analysis wrt standard epidemiology,
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    - ❖ $R_{\text{effective}} = r\ i\ S(t)/N(t)$,
    - ❖ $t \to R_{\text{effective}}(t)$ « decreasing outside shoot-topping » ?
  - Computing the asymptotic state in closed form (+∞ = winter) ?
    - ❖ without ontogenic resistance ($1/m = 0$ or $T = 0$),

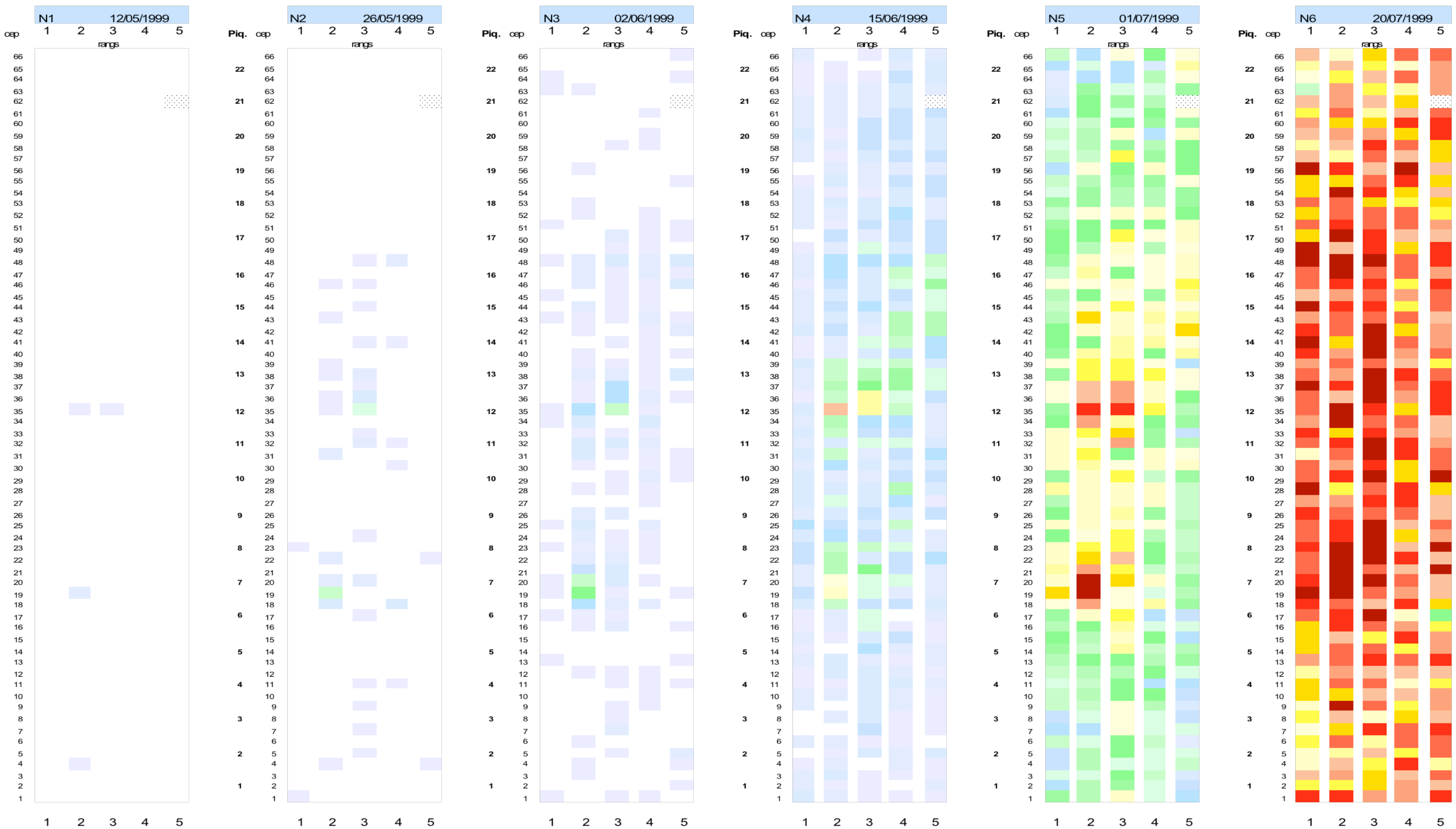
      $S = S^{infinity}$, $E = I = 0$, $R = R^{infinity}$.

    - ❖ with ontogenic resistance,

      $S = 0$, $E = I = 0$, $R = R^{infinity}$, $T = T^{infinity}$.

# Field data (1999), INRA Bordeaux

- Experimental plot with 5 rows & 66 vine stocks per row,



# *Spatially structured* plot - pathogen model

Pathogen dispersal :
**conceptual analysis.**

Long and short distance dispersal
type 1 : short alone,
type 2 : both (coalescence),
type 3 : (scaling effect)
reduced short / large long.

**Shigesada, N., Kawasaki. K.**

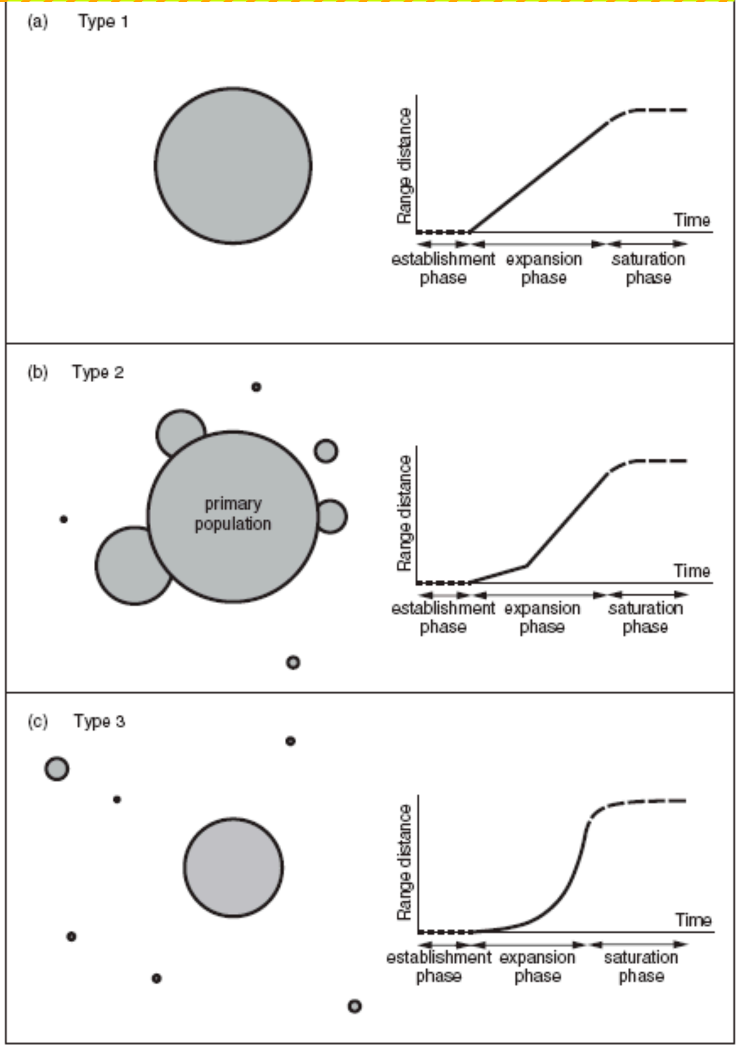
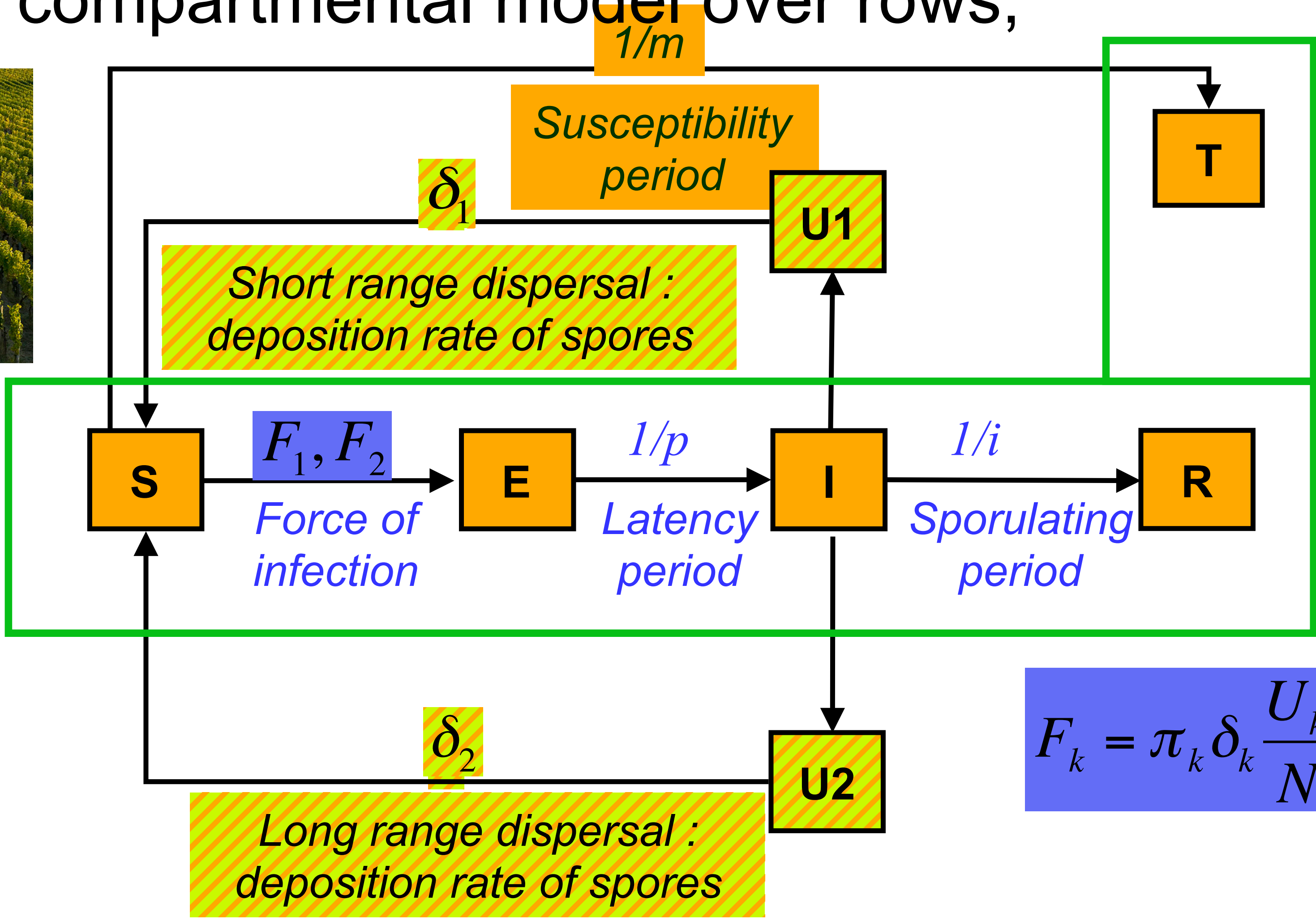



**Figure 17.1** Three types of range expansions: (a) type 1, (b) type 2, and (c) type 3. For each type, the spatial pattern and range-versus-time curve are shown on the left and right, respectively. See text for detail. (Adapted from Shigesada & Kawasaki 1997, with permission of Oxford University Press.)

# ***Spatially structured*** plot - pathogen model

- SEIRT compartmental model over rows,

*Host growth*




$$F_k = \pi_k \delta_k \frac{U_k}{N}$$

# ***Spatially structured*** plot - pathogen model

- **R**eaction-**D**iffusion / SEIRT model :

(1) Reaction-Diffusion system for dispersal of spores,

$$\begin{cases} \partial_t U_1 = \nabla \cdot (d_1(x,N) \nabla U_1) - \delta_1 U_1 + \gamma f(N) I, \\ \partial_t U_2 = \nabla \cdot (d_2(x,N) \nabla U_2) - V(x,t) \cdot \nabla U_2 - \delta_2 U_2 + \gamma (1 - f(N)) I. \end{cases}$$

❖ $U_1$ & $U_2$, short (plant scale) and long range dispersed spores,

- Fick's law diffusion, $0 < d_1(x,N) \leq d_2(x,N)$,
- Convection term $V(x,t)$, for average vs strong winds (?),

❖ $\delta_i$, deposition rate of spores $U_i$, $i=1,2$, $\delta_2 \leq \delta_1$,

❖ $\gamma$, production rate of spores per infectious unit,

❖ $f(N)$, % of spores dispersed at short range, Aylor (1999),

# ***Spatially structured*** plot - pathogen model

- **R**eaction-**D**iffusion / SEIRT model :

(2) SEIRT ODEs models over rows : production of spores by ***I*** and contamination of ***S*** by $U_1$ and $U_2$

$$\begin{cases} d_t S = \Lambda - (\pi_1 \delta_1 U_1 + \pi_2 \delta_2 U_2)\dfrac{S}{N} - \dfrac{1}{m} S \\ d_t E = +(\pi_1 \delta_1 U_1 + \pi_2 \delta_2 U_2)\dfrac{S}{N} - \dfrac{1}{p} E \\ d_t I = +\dfrac{1}{p} E - \dfrac{1}{i} I \\ d_t R = +\dfrac{1}{i} I \\ d_t T = +\dfrac{1}{m} S \end{cases}$$

$\pi_1 \gamma \approx r$, ODEs model,

$\pi_2 \leq \pi_1$, inoculum eff.

$$\Rightarrow d_t N = \Lambda = \alpha\left(1 - \frac{N}{K}\right) N.$$

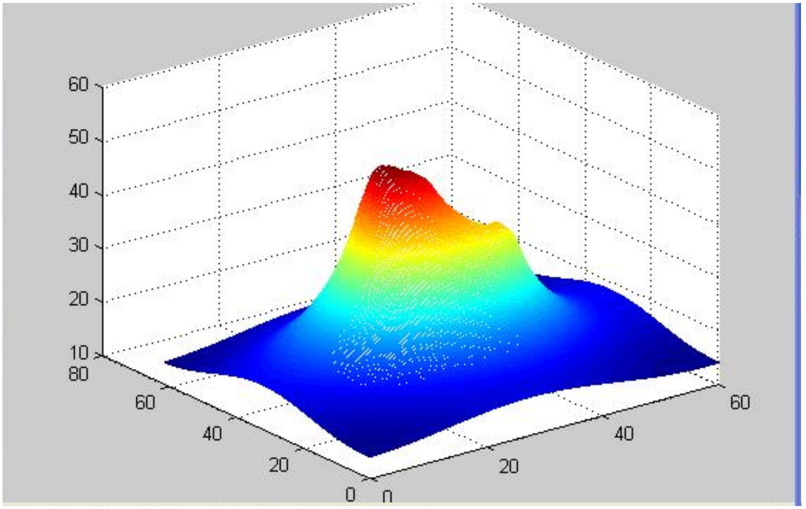
# ***Spatially structured*** plot - pathogen model (a)

- Basic model, R-D / SEIRT :
  - R-D system,
    - ❖ constant diffusivities, no convection term,
    - ❖ kinetic terms : homogeneous / heterogeneous,
    - ❖ cf. Zawolek & Zadoks (1992), Vanderplank (1963),
  - *Boundary conditions* ?
    - *homogeneous Dirichlet conditions* set on a much larger domain than the spatial range of interest,
    - Mathematical analysis : IBVP.
    - Numerical experiments (Matlab interface).

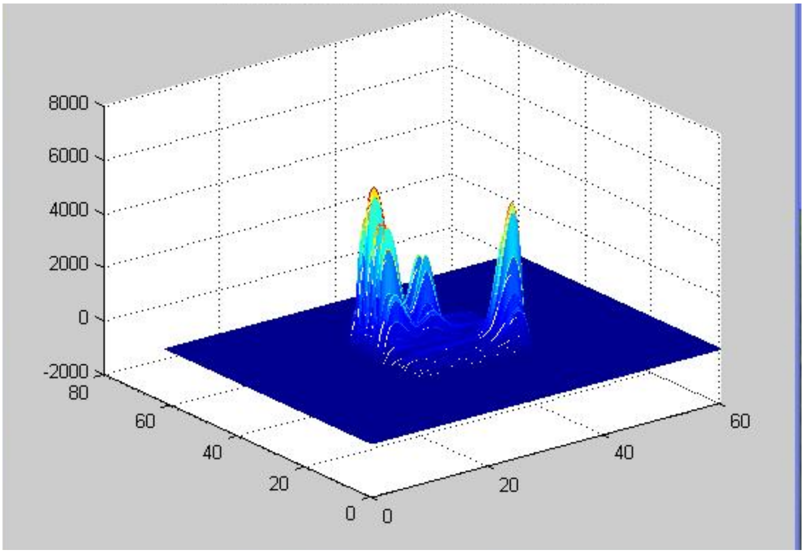
    vs
  - TW analysis.

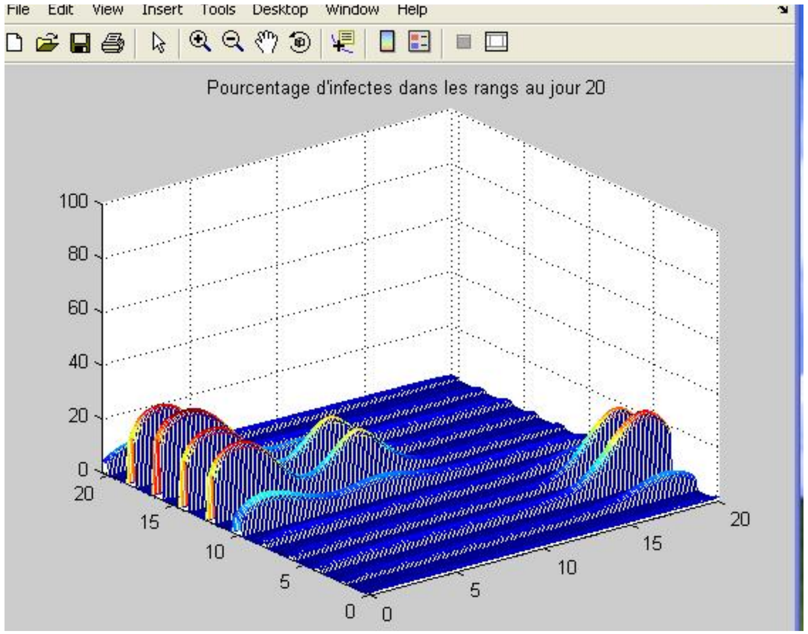
# ***Spatially structured*** plot - pathogen model (a)

Spores : long range dispersal




Spores : short range dispersal

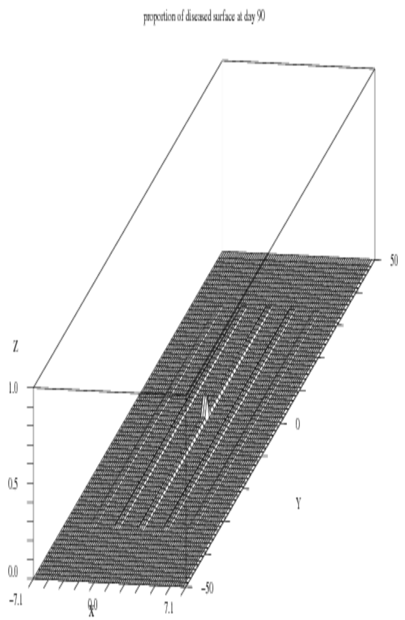


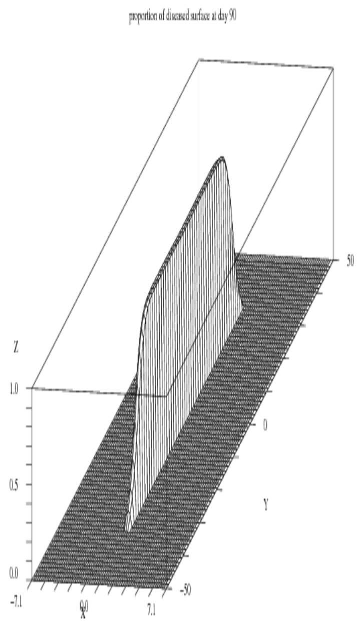



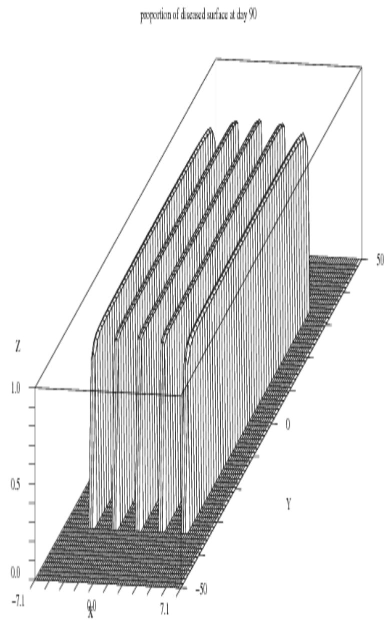

Spatial spread of infection

# ***Spatially structured*** plot - pathogen model (a)

- Proportion of disease lesion / dispersal, Day 90,
  - Long range dispersal only : weak uniform contamination,
  - Short range dispersal only : central row is contaminated,
  - Both : contamination spreads all over the five rows,



# ***Spatially structured*** plot - pathogen model (b)

- ***Qualitative analysis :***
  - Dimensionless model, R-D / SEIR (*no T*),

$$\frac{\partial U_1}{\partial t} = \Delta U_1 - \eta U_1 + \eta F I$$

$$\frac{\partial U_2}{\partial t} = d\Delta U_2 - \eta U_2 + \eta (1 - F) I$$

$$\frac{\partial S}{\partial t} = -R_0 ( U_1 + U_2 ) S$$

$$\frac{\partial E}{\partial t} = +R_0 (U_1 + U_2) S - \frac{1}{\tau} E$$

$$\frac{\partial I}{\partial t} = \frac{1}{\tau} E - I$$

# ***Spatially structured*** plot - pathogen model (b)

- ***Qualitative analysis* (*no T*) *:***
  - Wave fronts, $\varphi(x\text{-}c.t)$, 1D, pulses / infection, ***linking***,
    - ***Pre-infection state***,
      - ❖ $U_1 = U_2 = 0$, $S = K$, $E = I = R = 0$.
    - ***Post-infection state***,
      - ❖ $U_1 = U_2 = 0$, $S = K^{infinity}$, $E = I = 0$, $R = R^{infinity}$.
  - When $R_0 > 1$, minimal speed TW, $c*$,
    - implicit closed form ($K^{infinity}$),
    - $K^{infinity}$ a root of, $h = 1 - e^{-R_0 h}$

# *Spatially structured* plot - pathogen model (b)

- The value of c* is given by solving (F<1) the characteristic equation:

$$\text{Find } (\lambda^*, c^*) \text{ such that } 0 < \lambda^* < \frac{c + \sqrt{c^2 + 4d\eta}}{2d} \text{ and } \begin{cases} \psi(\lambda^*, c^*) = 1 \\ \dfrac{\partial \psi}{\partial \lambda}(\lambda^*, c^*) = 0 \end{cases}$$

with

$$\psi_{SEIR}(\lambda, c) = \frac{R_0}{(1 + c\lambda)(1 + c\tau\lambda)} \left( \frac{\eta F}{\eta + c\lambda - \lambda^2} + \frac{\eta(1 - F)}{\eta + c\lambda - d\lambda^2} \right)$$

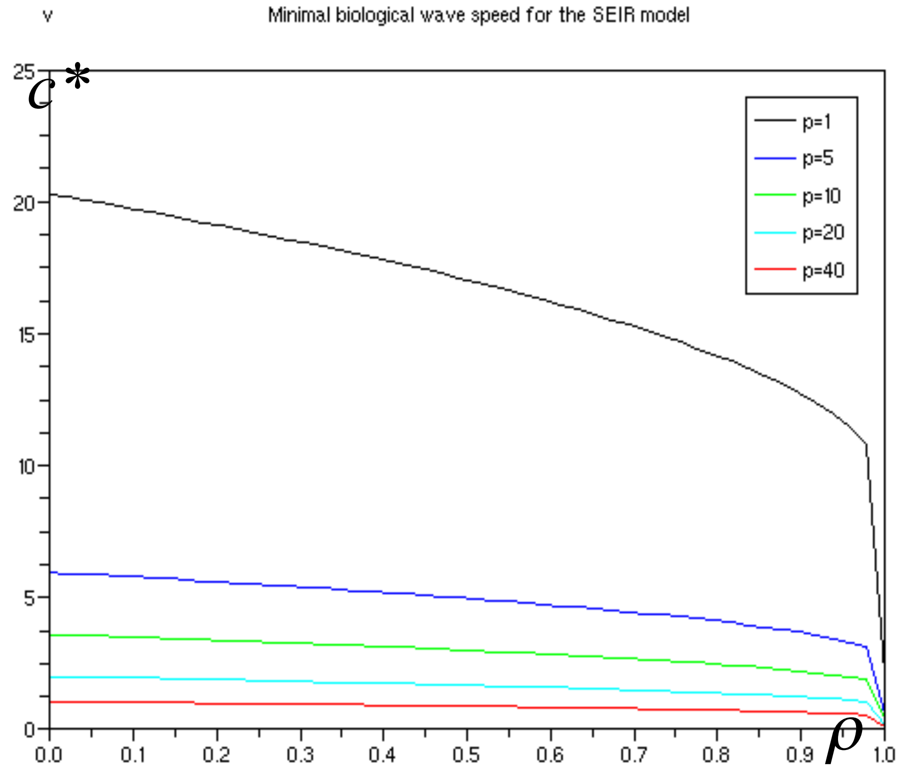
# ***Spatially structured*** plot - pathogen model (b)

- *Dimensionless minimal speed*, $c^*$,
  - Assuming : $\delta_2 = \delta_1$ , $\pi_2 = \pi_1$ ,
  - *F(N)*=$\rho$, % of short range dispersed spores,

Decreasing w.r.t. $\rho$.

Discontinuity at $\rho$ = *1.*

*cf. Shigesada & Kawasaki.*



# ***Spatially structured*** plot - pathogen model (b)

- *Dimensionless minimal speed*, $c^*$,
  - Assuming : $\delta_2 < \delta_1$ , $\pi_2 < \pi_1$ ,
  - *F(N)*=$\rho$, % of short range dispersed spores,

maximum achieved

at a suitable optimal $\rho$,

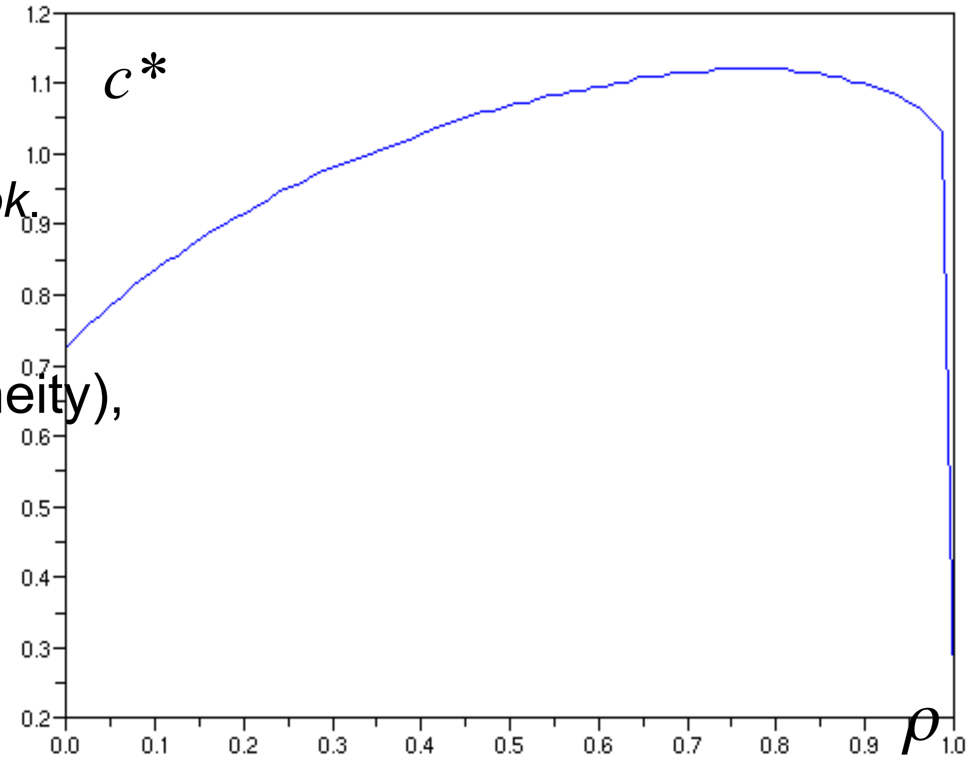
*cf. Zawolek & Zadok.*

actual speed (*slightly, 50%*)

over-estimated (?, heterogeneity),

Discontinuity at $\rho$ = *1.*

*cf. Shigesada & Kawasaki.*



# ***Spatially structured*** plot - pathogen model (b)

- 2D spatio-temporal periodical WF collaborative work A. Ducrot & H. Matano,

- ***in which direction is the speed faster ?***

  numerics : along rows,

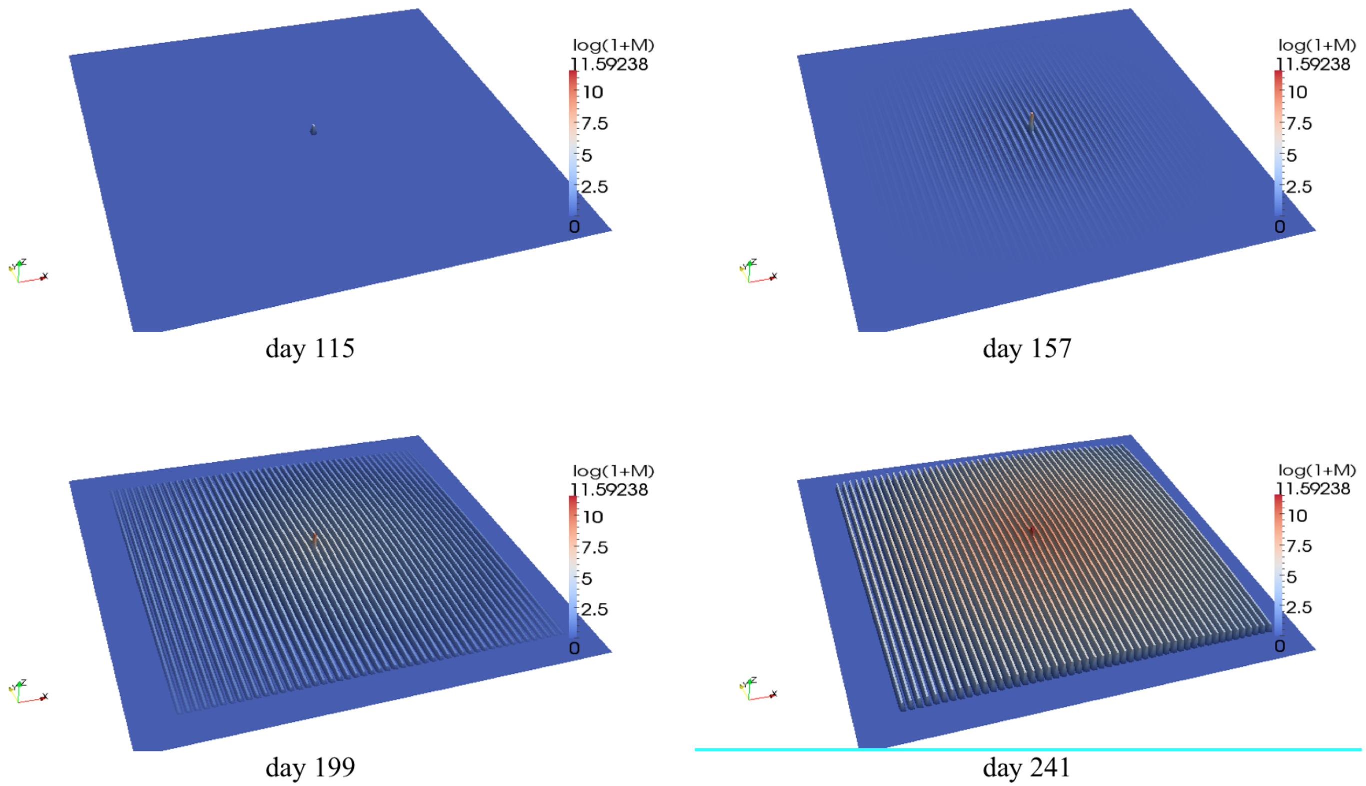
  vs

  data : across rows, but …

  – *Numerical work in progress (turbulence, nonlinear diffusion, architecture & $f(N)$, … ).*

# ***Spatially structured*** plot - pathogen model (b)

- 2D spatio-temporal dynamics,
- *Nonlinear short range dispersal, data fitting ?*



# Further developments …

- Collecting data and calibrating 2D and 3D models, including transport,
  - no known markers for *Erysiphe necator*, powdery mildew, compared to pollen dispersal,
  - Secondary infections from alien sources ...
- Control problems using fungicides,
  - when ?
  - coupling with shoot topping driving host dynamics ?
  - reducing or delaying the pathogen speed of propagation if coupled to ontogenic resistance ?

# ***Spatially structured*** plot - pathogen model (c)

- Spatially periodic spatial plot,
  - reference cell, $Y$, "a single plant" (short range scale),
  - small parameter, $\varepsilon > 0$,
  - spatial domain, $\Omega$, a set of cells of size $\varepsilon * Y$,
  - available vegetal tissue density, function $\chi$, period $Y$, $\chi(x,t,x/\varepsilon)$.

- Simpler to handle as $\varepsilon \to 0$ ?
  - spatially periodic: ill-conditioned

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  - M1 2009 et M2R 2010, 2011.