Nick Britton
University of Bath

Title of Presentation: Travelling Waves in Systems of Integrodifference Equations

Abstract: Integro-difference equations of the form

$$N^{t+1}(x) = \int_{-\infty}^{\infty} k(x-y)R(N^t(y))N^t(y)dy$$

(or the equivalent in higher dimensions) have recently been proposed as a model for certain biological populations. The underlying assumptions are that the population has non-overlapping generations split into a growth phase, with dynamics determined by the reproductive ratio R, and a dispersal phase, with dispersal kernel k. A typical example would be an annual plant with local fertilisation and seed dispersal determined by k. If R is constant this equation admits exponential travelling wave solutions that may invade virgin territory, and it has been conjectured that solutions of the nonlinear problem behave like those of its linearisation about zero. In this contribution we shall generalise such results to systems of two integro-difference equations of the form

$$N_1^{t+1}(x) = \textstyle\int_{-\infty}^{\infty} k_1(x-y)R_1(N_1^t(y), N_2^t(y))N_1^t(y)dy,$$

$$N_2^{t+1}(x) = \textstyle\int_{-\infty}^{\infty} k_2(x-y)R_2(N_1^t(y), N_2^t(y))N_2^t(y)dy.$$

Here N_1 and N_2 are two types; they could be two biological species or two genotypes or phenotypes of the same species.

In the absence of spatial effects, the dynamics are determined by the equations

$$N_1^{t+1} = R_1(N_1^t, N_2^t)N_1^t, \quad N_2^{t+1} = R_2(N_1^t, N_2^t)N_2^t,$$

where R_1 and R_2 are the reproductive ratios of types 1 and 2 respectively. We shall restrict attention to the case where the reproductive ratios are frequency- but not density-dependent, so that the R_i depend on the proportion of each type in the population but not on the absolute numbers of each. This is similar to the assumption that R is constant which is made in most rigorous studies of the single equation, and which results there in a linear equation, but here it does not result in a linear system.

Guided by the results for a single equation, we shall look for travelling wave solutions of our system of equations, representing either a wave of fixation of one type or the other or a wave of invasion of one or both types into virgin territory.